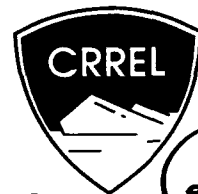


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## Energy Absorption of Graphite/Epoxy Plates Using Hopkinson Bar Impact

Piyush K. Dutta, David Hui and Magna R. Altamirano

October 1991



*For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380, Metric Practice Guide, published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.*

# CRREL Report 91-20



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## PREFACE

This report was prepared by Dr. Piyush K. Dutta, Materials Research Engineer of the Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory (USACRREL); Dr. David Hui, Associate Professor of Mechanical Engineering, University of New Orleans; and Magna R. Altamirano, graduate student, University of New Orleans. The work has been carried out in the Materials Research Laboratory of CRREL. Funding for this work was provided partially by DA Project 4A762784AT42, *Cold Regions Technology*, Task SS, Work Unit 019, *Behavior of Materials at Low Temperatures*, and partially by Department of the Air Force Flight Dynamics Laboratory at Wright Patterson AFB, Ohio, under MIPR FY1456-90-N0066 of July 1990.

The authors express their appreciation to Dr. Arnold H. Mayer, Assistant for Research and Technology, AFWAL Flight Dynamics Laboratory, for his support, ideas, suggestions, and ballistic impact data analysis results. The authors are especially indebted to Gregory Czarnecki, Scientist, AFWAL Flight Dynamics Laboratory, for supplying the specimens and closely working with the authors to support the experimental tasks. John Kalafut of the Engineering and Measurement Services Office of CRREL provided much-needed support and participated in the experimental measurement program. Capt. Karen Faran of CRREL did the arduous task of numerical analysis of the data. Special thanks are given to Frederick Gernhard of the CRREL machine shop who built the test fixture.

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# Energy Absorption of Graphite/Epoxy Plates Using Hopkinson Bar Impact

PIYUSH K. DUTTA, DAVID HUI AND MAGNA R. ALTAMIRANO

## INTRODUCTION

Most literature dealing with energy absorption in laminated composites describes the use of a dropweight or pendulum impact test apparatus. In these techniques, energy absorption is determined by subtracting the residual kinetic energy from the initial energy. Thus the effects of stress wave propagation, which is a source of damage initiation, cannot be examined using these testing techniques. Further, the laminate's energy absorption during the penetration process and the projectile's velocity, contact force, and duration of impact are difficult to measure. Hopkinson bar testing eliminates these restrictions and allows a thorough examination of the projectile impact process.

The effort described in this report involved an analytical and experimental study of energy absorption within graphite/epoxy plates impacted by a hemispherical-nosed impactor attached to a split Hopkinson bar test apparatus. The work is novel in that the plate's energy absorption was accurately measured using the recorded incident, reflected, and transmitted stress waves. The Hopkinson bar test apparatus allowed accurate determination of velocity, force, and energy absorption information during the entire impact duration. Thus the setup allowed "controlled" testing, capable of determining the energy absorption of impacted composite specimens. The only limitation was that the impactor was forced into the laminate at relatively low velocities.

## MECHANICS OF WAVE PROPAGATION

The theoretical wave mechanics involved in the impact process are discussed based on elementary principles of unidirectional stress wave propagation. The

duration of impact produced by the striker bar on the Hopkinson bar apparatus persists for a few hundred microseconds, and it takes a small but finite time for the effects of the impact force (stress wave) to enter the graphite epoxy test laminate at the opposite end of the bar. Upon entering the laminate, a portion of the incident stress wave is propagated into the transmitter bar and another portion is reflected back into the incident bar. Incident, reflected, and transmitted stress waves are measured by strain gauge instruments described elsewhere (Dutta et al. 1987).

In the current study, the laminate specimens consisted of 30 plies of AS4/3502 graphite epoxy with the following stacking sequence:

$$[(\pm 45/0_2)_2/90/\pm 45/0_2/\pm 45]_s$$

The indenter was driven into the laminate by the force of the incident bar's stress wave. Figure 1 shows a schematic of the Hopkinson bar, the impactor, and the laminate panel clamping assembly. Figure 2 is a photograph of the setup. The transmitter bar is free to slide forward or backward. As the striker hits the incident bar and drives the indenter into the laminate plate, the plate bends and behaves like a spring-mass system. Plate bending was recorded from strain gauges mounted on the rear face of the laminate.

The wave mechanics within this system was analyzed to determine the force-displacement relationship between the impactor and the composite laminate. Energy absorbed during the process was also determined as a function of time. The velocity, force, and energy absorbed were calculated from incident, reflected, and transmitted waveforms.

An impact of bar 1 (the striker bar) into bar 2 (the incident bar) produces a compressive force at interface A. According to elementary stress wave propagation

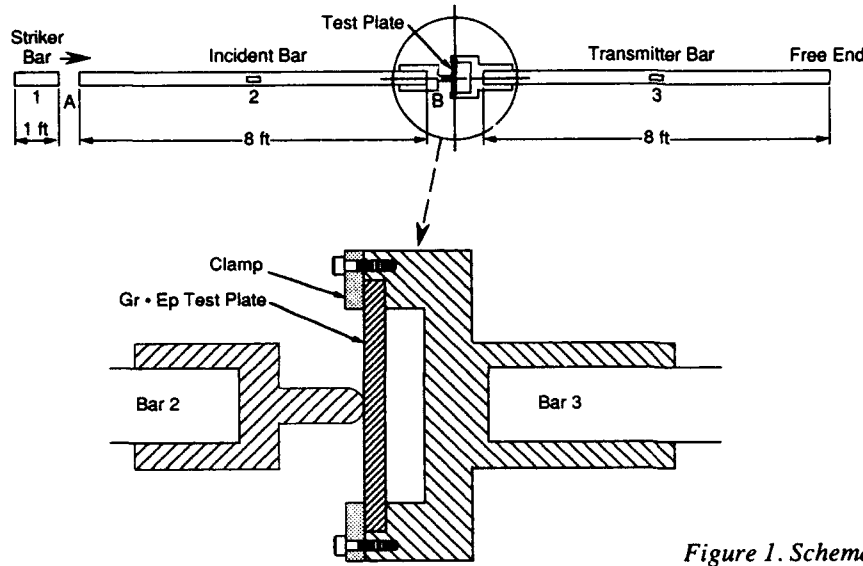


Figure 1. Schematic of the Hopkinson bar test setup.

theory (Kolsky 1963, Dutta et al. 1987), stress  $\sigma$  and force  $P$  generated by the impact are

$$\sigma = (\rho/g) c V \quad (1)$$

and

$$P = (A) (\rho/g) (cV) \quad (2)$$

where  $\rho$  is the density of the incident bar,  $g$  is the acceleration due to gravity,  $c$  is the speed of sound in the bar,  $V$  is the particle velocity in bar 2, and  $A$  is the cross-sectional area. The stress wave propagates along bar 2 to the opposite end where it encounters a discontinuity of its impedance (a product of density and stress wave velocity,  $\rho c$ ) at interface B. In the experimental setup, the impactor and laminate respond to this incoming stress pulse. As a result of the discontinuity, a portion of the stress pulse energy ( $E_I$ ) is reflected ( $E_R$ ) and a portion is transmitted ( $E_T$ ) through the spring-mass system. Most of the remaining energy goes into the formation of plastic deformation of the plate. Some energy is also lost in overcoming frictional forces in various joints and through noise and vibration of the entire system. Net energy lost in the impact can be calculated by subtracting the energy of the transmitted and reflected waves from that of the incident wave:

$$E_A = E_I - E_R - E_T \quad (3)$$

The procedure for computing energy from the stress vs time waveform is given in Appendix A.

The force of penetration at any time  $t$  can be calculated more directly by considering the particle velocity in the following equation:

$$V = du/dt = \frac{\sigma}{\left(\frac{\rho}{g}\right) c} \quad (4)$$

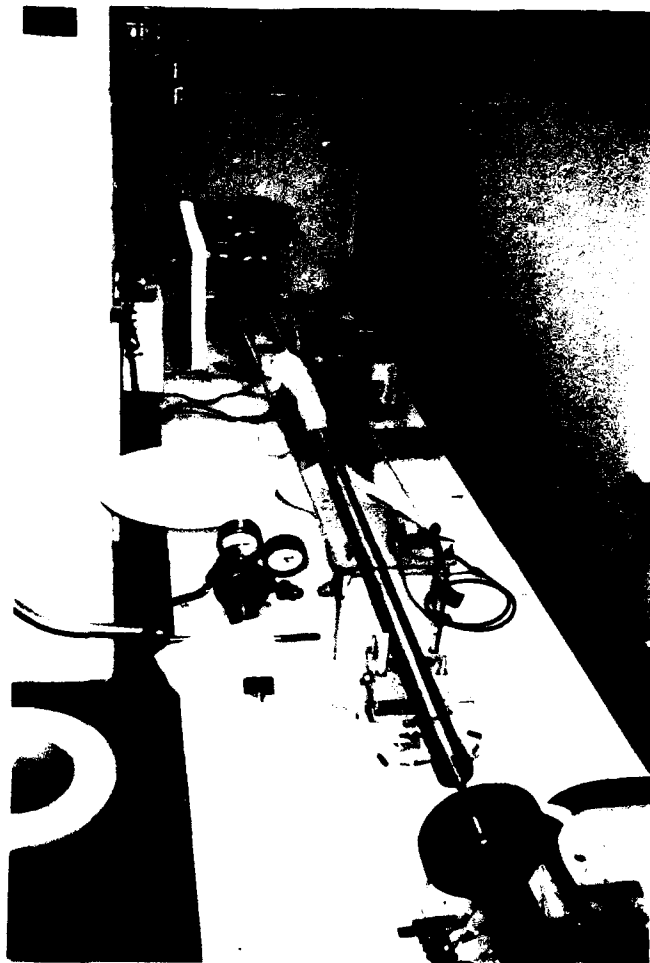
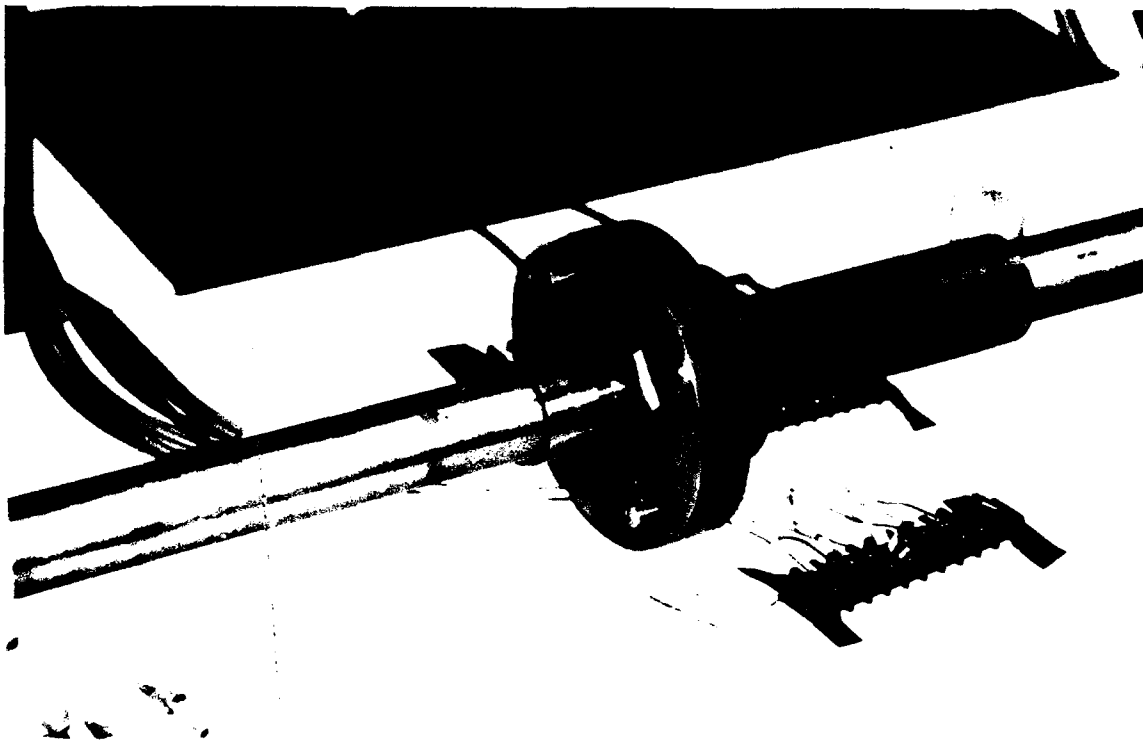
where  $u$  is the axial displacement of the bar and is given by

$$u = \frac{1}{\left(\frac{\rho}{g}\right) c} \int_0^t \sigma(t) dt \quad (5)$$

Under the incident stress wave  $\sigma_i(t)$ , the penetrator end of bar 2 will be subjected to a displacement  $u_i$  given by

$$u_i = \frac{1}{\left(\frac{\rho}{g}\right) c} \int_0^t \sigma_i(t) dt \quad (6)$$

Due to the change of impedance, a portion of the stress wave is reflected, which produces a corresponding displacement of the indenter in the opposite direction; this displacement  $u_r$  is



*Figure 2. Photograph of the setup showing the hemispherical indenter.*

$$u_r = \frac{1}{\left(\frac{\rho}{g}\right)^c} \int_{t=0}^t \sigma_r(t) dt \quad (7)$$

The resultant displacement  $u_t$  is given by

$$u_t(t) = \frac{1}{\left(\frac{\rho}{g}\right)^c} \int_{t=0}^t [\sigma_i(t) - \sigma_r(t)] dt \quad (8)$$

Force on the penetrator  $[F(t)]$  is the algebraic sum of the incident force  $A\sigma_i(t)$  and the reflected force  $A\sigma_r(t)$ :

$$F(t) = A \int_{t=0}^t \{[\sigma_i(t) + \sigma_r(t)]\} dt \quad (9)$$

From eq 8 and 9, the force-displacement curve for the indentation can be plotted. The area under the curve will yield the energy absorbed as a function of time throughout the impact.

Energy absorbed by the graphite epoxy plate can be computed from the following energy balance equation (Greszczuk 1982, Shivakumar et al. 1984):

$$E_A = E_c + E_b + E_m + E_d + E_f \quad (10)$$

where  $E_c$  = the energy expended in Hertzian contact indentation,

$E_b$  = the energy due to plate bending,

$E_m$  = the energy due to the stretching of the middle plate surface,

$E_d$  = the energy due to permanent plate damage, and

$E_f$  = the energy due to friction, heating, etc.

All of these energies are recoverable, except the damage and frictional energy. Plate damage may include any or all of the following modes of failure: matrix cracking, delamination, fiber breakage, and spalling.

The bending energy of a circular plate clamped along its edge under a concentrated load  $P$  is (Roark and Young 1975, Greszczuk 1982)

$$E_b = (P^2) \left( \frac{3a^2}{2} \right) (1 - \nu_r^2) (4\pi E_r h^2)^{-1} \quad (11)$$

where  $a$  is the radius of the circular plate,  $h$  is the thickness of the plate, and  $E_r$  and  $\nu_r$  are the Young's modulus and Poisson's ratio, respectively, of the plate

material in the in-plane radial direction. Since the deflection was measured to be less than 20% of the laminate thickness, the membrane energy ( $E_m$ ) was neglected.

Although Hertzian contact law was established for local static indentation of two elastic bodies, it is commonly applied to impact situations where permanent plastic deformations occur (Goldsmith 1960). The Hertzian contact energy of a spherical indenter with radius  $R$  impacting a flat plate is (Greszczuk 1982)

$$E_c = (P^{5/3}) (4/5) (3\pi/16)^{2/3} [ (k_1 + k_2) / (R/2)^{1/2} ]^{2/3} \quad (12)$$

where

$$k_1 = (1 - \nu_s^2) / (\pi E_s) \quad (13)$$

$$k_2 = (1 - \nu_r^2) / (\pi E_r) \quad (14)$$

where  $E_s$  and  $\nu_s$  are the Young's modulus and Poisson's ratio of the steel indenter, respectively,  $E_r$  is the laminate's modulus in the thickness direction, and  $\nu_r$  is the laminate's Poisson's ratio.

Assuming point contact between the indenter and plate, the deflection can be computed from the in-plane stress  $\sigma$  (Roark and Young 1975):

$$\sigma h^2/6 = [P/(4\pi)] (1 + \nu) \{ \ln [a/(0.325h)] \} \quad (15)$$

Further, deflection  $y$  at the center of the circular plate due to a concentrated load  $P$  is

$$y = (Pa^2/\pi) (3/4) [(1 - \nu^2)/(Eh^3)] \quad (16)$$

Measured strain can be converted to stress by multiplying the strain by the in-plane Young's modulus, so  $\sigma$  is a known quantity. Eliminating the applied load  $P$  from the above two equations, the deflection can be obtained in the form

$$y = \frac{a^2 \sigma (1 - \nu)}{(2hE) \ln [a/(0.325h)]} \quad (17)$$

Upon stress wave arrival, the indenter accelerates into the plate and creates a compressive stress wave that propagates through the thickness. Stress wave velocity

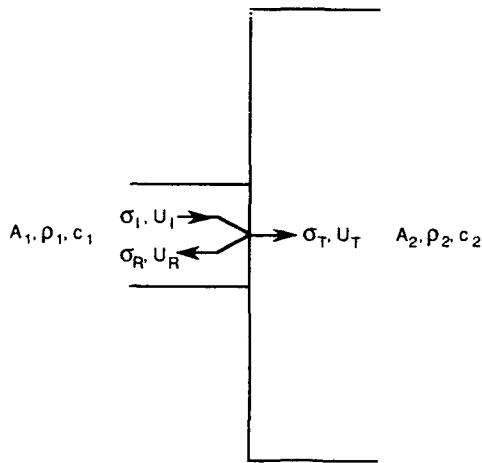


Figure 3. Stress pulse reflections and transmissions through the interface.

in the thickness direction of the plate is approximately (McMaster 1963)

$$c_z = \left[ E_z / (\rho/g)^{1/2} f(v) \right] \quad (18)$$

$$f(v) = \left\{ (1-v) / [(1+v)(1-2v)] \right\}^{1/2} \quad (19)$$

where  $E_z$  is Young's modulus in the thickness direction and  $v$  is Poisson's ratio (for  $n = 0.31$ ,  $f(v) = 1.17732$ ).

For a quasi-isotropic graphite epoxy laminate, where  $E_r = 51.16$  GPa (7.42 Msi),  $E_z = 11.86$  GPa (1.72 Msi), and  $\rho = 1522.4$  kg m<sup>-3</sup> (0.055 lb-mass in.<sup>-3</sup>), wave speed  $c_z = 3.043$  km s<sup>-1</sup> ( $0.1198 \times 10^6$  in. s<sup>-1</sup>). This wave will propagate with speed  $c_z$  as a compressive wave and will eventually reflect off the rear (free) surface as a tensile wave with the same velocity. When the wave arrives at the rear surface, the plate moves away from the indenter

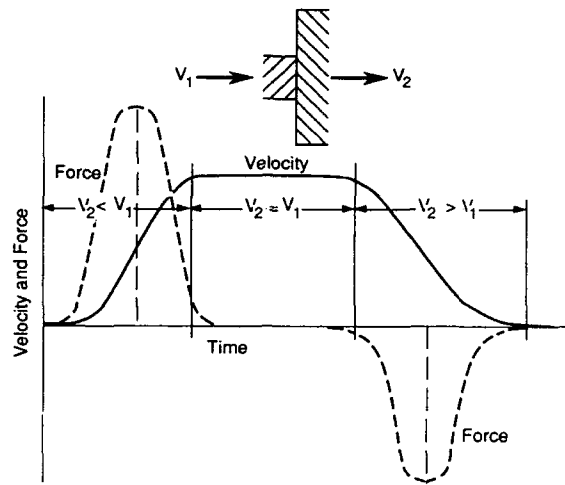


Figure 4. Interface velocity and force.

at velocity  $V_1$  while the indenter continues to move into the plate with a velocity  $V_1$  (see Fig. 3).

During the duration of the stress wave, three conditions are possible. First, when  $V_2 < V_1$ , the indenter accelerates into the plate (positive contact force) with a force vs time relationship as shown in Figure 4. Second, when  $V_2 = V_1$ , both the indenter and the plate move in the incident direction with no change in contact force. Third, when  $V_2 > V_1$ , loss of contact occurs between the indenter and plate, and a tensile force develops in the incident bar. Both the indenter and the plate move forward monotonically even though loss of contact occurs.

## DISCUSSION OF RESULTS

Figure 5 is a graph of energy absorption vs displacement. The energy absorption of the plate (curve A) is

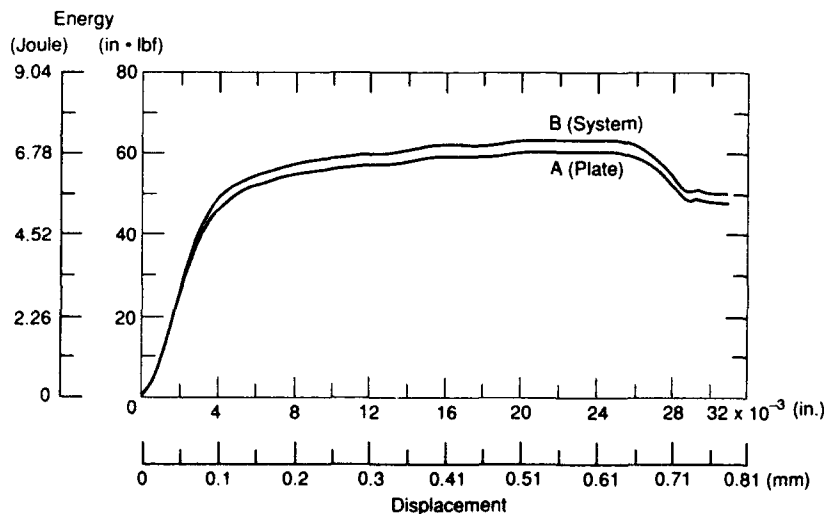


Figure 5. Impact energy absorption results.

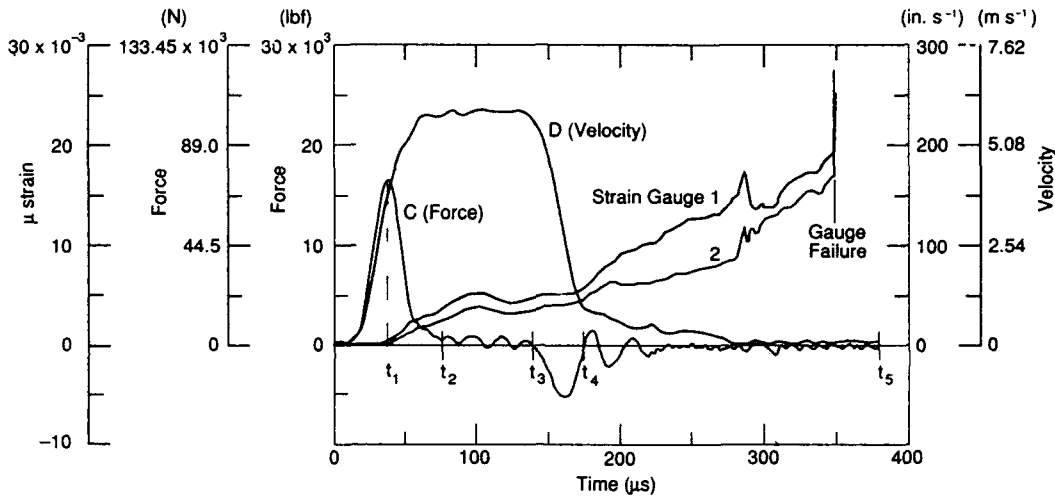


Figure 6. Results of force, velocity, and strain history measurements.

computed from the force vs displacement of the indenter. Energy absorbed in the system, consisting of the plate and clamping fixture shown in curve B, is computed from the incident, reflected, and transmitted waves. Appropriate formulae are given in Appendix A. The small difference between these two energy absorptions is the energy loss in the fixture ( $E_p$ ).

Indenter force vs time (curve C of Figure 6) follows the model of Figure 4. There are five stages involved in the penetration process:

- In the initial loading stage ( $0 < t < t_1$ ), the indenter accelerates into the plate, as can be seen from the increasing slope of the velocity trace (curve D).
- The second stage ( $t_1 < t < t_2$ ) is the initial unloading stage where the indenter force decreases rapidly and the corresponding velocity increases at a slower rate. The drop in the indenter force is attributed to damage either in the contact region or near the rear surface due to a reflected tensile wave. The latter may cause the plate to move away from the indenter, thus reducing the contact force (which will be more evident in the next stage).
- The third stage is the constant velocity stage ( $t_2 < t < t_3$ ) in which there is very little contact force. Referring to eq 9, note that at this stage  $\sigma_I(t) = \sigma_R(t)$ , indicating that there is no interfacial contact force.
- In the fourth stage ( $t_3 < t < t_4$ ), the effects of stage 3 become more pronounced, and separation between the indenter and plate is possible. Similar loss of contact behavior has been reported by Wu and Springer (1988) and Sun and Chen (1985). At this stage, the applied incident stress amplitude begins to di-

minish but, because of the gap between the plate and the indenter, the indenter experiences a tensile force. (Note a reversal of the force direction.)

- The final stage ( $t_4 < t < t_5$ ) is the vibration stage, during which the plate exerts an oscillating force on the indenter where contact may be lost and regained many times.

Figure 7 shows the cumulative energy absorption vs time. Energy absorbed increases rapidly with time in the first two stages and very slowly in the third stage. During the plate separation stage, a portion of the energy returns to the incident bar as the indenter accelerates into the free space in front of it. This energy is subtracted from the cumulative energy, so the energy curve dips at this stage. Energy absorbed in the final stage is constant and is the energy required to cause plate damage.

Figure 6 also shows data from two strain gauges mounted at two positions on the rear surface. Strain gauge 1 is mounted at the plate center and gauge 2 near the corner. The strain gauge data show that plate bending continually increases but remains small during the first four stages of impact. The increase becomes more pronounced in the final vibration stage and results in strain gauge failure near  $t = t_5$ .

Figure 8 shows the laminate's energy absorption  $E_A$  vs the maximum velocity of the indenter. The data follow the relationship

$$E_A = n V^m \quad (20)$$

where exponent  $m$  is found to be 2.55 for room temperature (21°C) and 4.53 for low temperature (−30°C). The value of the constant  $n$  for 21°C is  $3.75 \times 10^{-5}$  and for −30°C is  $1.01 \times 10^{-9}$ .

These data are currently considered a precursor to a

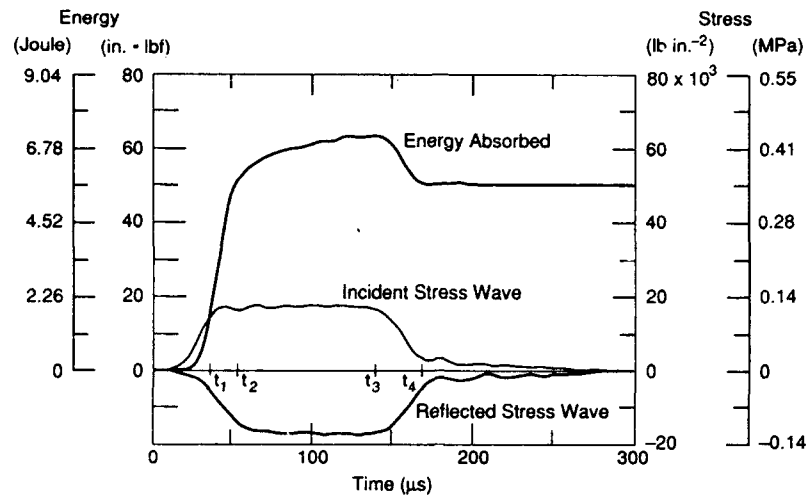


Figure 7. Bar stresses and energy absorption history.

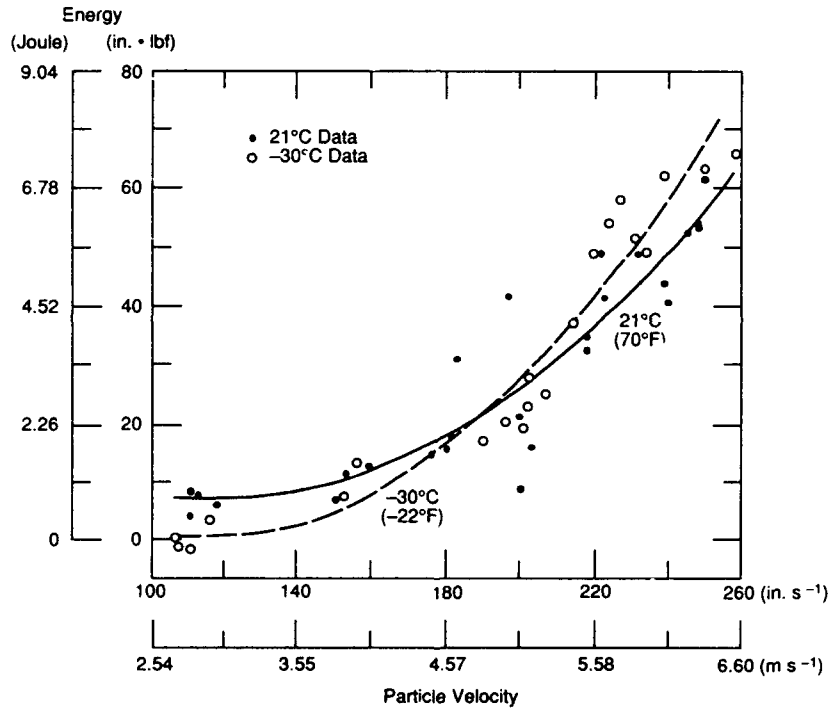


Figure 8. Influence of temperature and velocity on energy absorption.

detailed analysis of temperature's influence on the damage mechanisms associated with composite plate impact. It is well known that low temperature reduces the fracture toughness of most materials, but no extensive data are available for composites. In research performed by Nishijima and Okada (1982), cloth-reinforced epoxide resin composites were subjected to Charpy impact tests at room and liquid nitrogen temperatures. These tests demonstrated that a reduction in temperature causes a reduction in impact strength. Therefore, for a given

impact velocity more damage occurs at lower temperatures and more energy is absorbed. The experimental data in Figure 8 clearly show this trend after a transition velocity of about  $4.8 \text{ m s}^{-1}$  ( $190 \text{ in. s}^{-1}$ ). Above this velocity, more energy is absorbed by specimens held at low temperature. Below this transition velocity, the low temperature has an opposite effect, as discussed next.

Greszczuk (1982) and Shivakumar et al. (1985) have shown that in the low velocity range, among non-recoverable energies, the Hertzian contact force is the

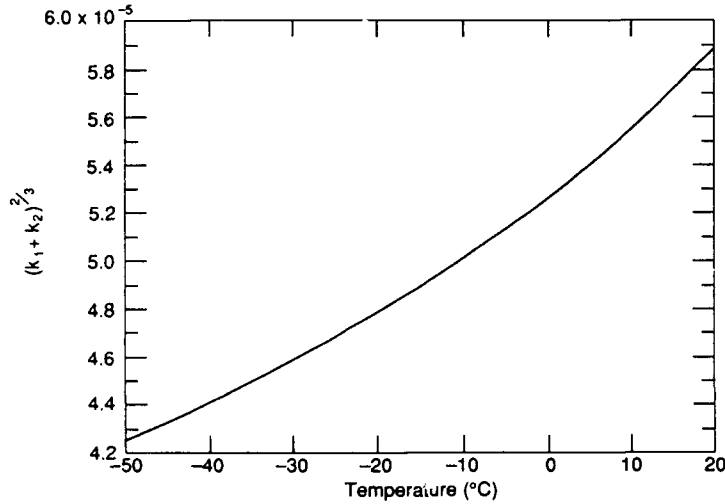


Figure 9. Effect of temperature on Hertzian contact energy.

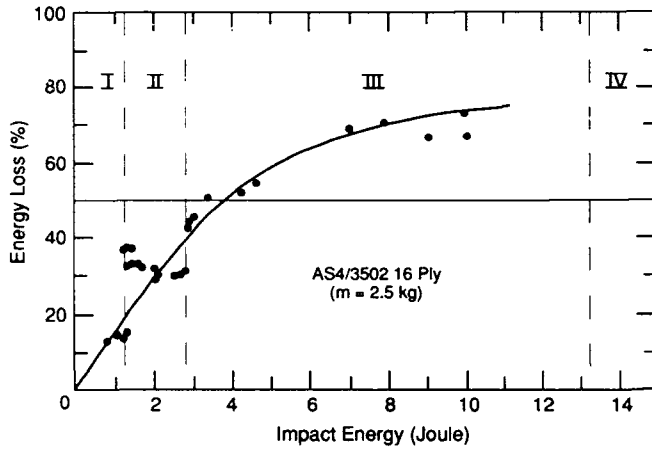


Figure 10. Energy loss as a function of impact energy (after Sjoblom et al. 1988).

most dominating energy absorption mechanism. From eq 12 this energy is a function of  $(k_1 + k_2)^{2/3}$ . Substituting  $\nu_s = 0.3$  and  $E_s = 206.9$  GPa ( $30 \times 10^6$  psi) for a steel indenter into eq 13,  $k_1 = 0.967 \times 10^{-9}$ . Referring to eq 14,  $k_2$  contains the term  $E_s$  (the Young's modulus of the composite plate in the thickness direction), which is influenced at low temperature by the change in elastic modulus  $E_m$  of the resin matrix. The empirical relation established by Dutta (1989) for matrix stiffening with lower temperature is

$$E_m(T) = E_m(T_0) - 2.9 \times 10^3 (T - T_0) \quad (21)$$

where  $T_0$  is the room temperature and  $T$  is the low temperature in °C. Applying the above relationship to the rule-of-mixtures equation (Tsai and Hahn 1980)

$$E = \frac{E_f E_m}{V_f E_m + V_m E_f} \quad (22)$$

yields

$$E_z = \frac{E_f E_m(T)}{\nu_f(T) E_m(T) + \nu_m E_f} \quad (23)$$

where  $\nu_m$  and  $\nu_f$  are the matrix and fiber volume ratios, and  $E_m$  and  $E_f$  are the matrix and fiber modulus of elasticity.  $(T)$  indicates the prevailing temperature of the matrix. Solving for  $E_z$  and setting  $\nu_f = 0.31$ ,  $k_2$  is determined via eq 14. Figure 9 shows a plot of  $(k_1 + k_2)^{2/3}$  vs temperature. It is clear from this graph that as the temperature is reduced the value of  $E_c$  (the Hertzian contact energy, which is a direct function of  $(k_1 + k_2)^{2/3}$ ) decreases. The experimental results in Figure 8 confirm this.

In a separate ballistic experiment involving the penetration of identically configured 30-ply graphite epoxy laminates by 12.7 mm (0.5 in.) spheres, Mayer and Altamirano\* observed energy absorption  $E_a$  in the form

\* Personal communication (1990) from A. Mayer, Air Force Wright Laboratories, Dayton, Ohio.

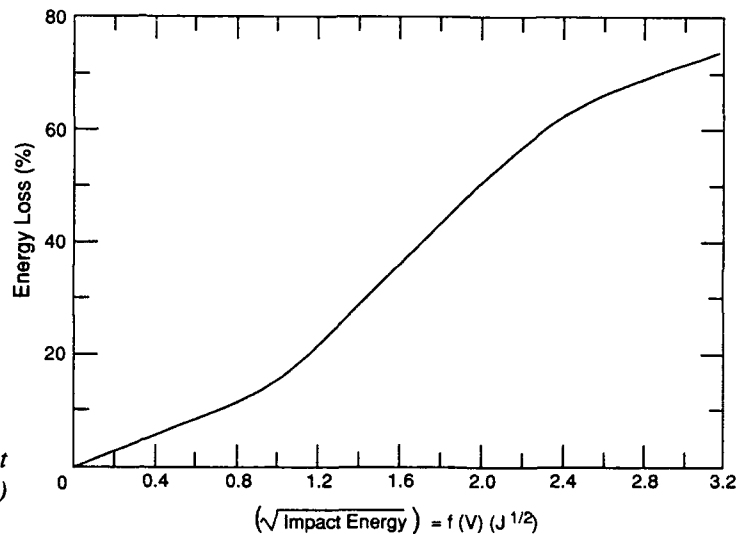


Figure 11. Energy loss as a function of impact velocity as obtained from Sjoblom et al. (1988) data.

$$E_a = C_0 V^{1.489} \quad (24)$$

where  $V$  is the impact velocity and  $C_0$  is a constant. Experimental velocities ranged from 500 to 1500 m s<sup>-1</sup> (1600–4900 ft s<sup>-1</sup>). Comparing the exponents of  $V$  from the three series of experiments—1.489 from the ballistic tests, 2.55 from the Hopkinson bar room-temperature tests, and 4.53 from the Hopkinson bar low-temperature tests—it is clear that the damage mechanisms and energy partitioning are different at higher velocities than under Hopkinson bar low-velocity impact. The lower value of the exponent suggests a failure mechanism that progressively requires less energy as the velocity is increased. This trend was also validated by Sjoblom et al. (1988) (Fig. 10). The solid line in this figure shows that energy loss is a function of  $V^2$ . A reconstruction of the energy loss vs velocity (Fig. 11) clearly shows a decrease in the energy absorption rate at higher velocities.

## CONCLUSIONS

The energy absorption of laminated plates under normal impact generated by a Hopkinson bar apparatus was examined. Using wave propagation theory, the energy absorption agrees with theoretical predictions. The force, velocity, and energy were recorded throughout the impact event, which encompassed several hundred microseconds. The experimental procedure provided a controlled condition enabling the full analysis of events during this period. Loss of contact between the impactor and laminate occurred frequently during the impact process due to stress waves in the plate reflecting back and forth through the thickness. The energy balance equation provides a powerful tool for analyzing the energy available to cause laminate damage.

Low temperatures influence the energy absorption characteristics and control the damage mechanisms. Existence of a transition velocity was observed; above this velocity energy absorption from an impact is higher than at room temperature. At low temperature, reduced impact strength causes a larger volume of damage. Below the transition velocity (at low temperatures) the laminate absorbs less energy, which can be approximated by Hertzian contact theory.

When comparing data generated via the Hopkinson bar apparatus to data obtained via high velocity ballistic impact, the failure mechanism is seen to be a function of impact velocity. As the impact velocity increases, the fracture requires less energy.

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## APPENDIX A: WAVE PROPAGATION IN A HOPKINSON BAR APPARATUS

The incident, reflected, and transmitted energy are denoted by  $E_I$ ,  $E_R$ , and  $E_T$  respectively. The governing equations for stress  $\sigma(t)$ , velocity  $V(t)$ , and displacement  $u(t)$  are

$$\sigma(t) = (\rho)(c/g)V(t) \quad (A1)$$

$$V(t) = du/dt \quad (A2)$$

$$u(t) = (c/E) \int_{t=0}^t \sigma(t) dt \quad (A3)$$

where  $E$  = Young's modulus,

$t$  = time,

$\rho$  = density,

$V(t)$  = velocity,

$c$  = the wave speed in the material, and

$g$  = the acceleration due to gravity.

Further, the energy of the wave in a one-dimensional bar is

$$E = \frac{Ac}{E} \int_{t=0}^t \sigma^2(t) dt \quad (A4)$$

where  $A$  is the cross-sectional area of the circular Hopkinson bar. Displacement of the indenter  $u_I(t)$  can be obtained from the incident, reflected, and transmitted stresses ( $\sigma_I(t)$ ,  $\sigma_R(t)$ , and  $\sigma_T(t)$ , respectively):

$$u_I(t) = (c/E) \int_{t=0}^t [\sigma_I(t) - \sigma_R(t) - \sigma_T(t)] dt \quad (A5)$$

The force on the indenter is

$$F(t) = (A) [\sigma_I(t) + \sigma_R(t)] \quad (A6)$$

Energy produced by the indenter ( $E_p$ ) can be computed from an integration of the product of force and displacement over the duration of the wave:

$$E_p = (Ac/E) \int_{t=0}^t \{ [\sigma_I(t) + \sigma_R(t)] - [\sigma_I(t) - \sigma_R(t) - \sigma_T(t)] \} dt \quad (A7)$$

The energy of the incident, reflected, and transmitted waves can then be computed from

$$(E_I, E_R, E_T) = (Ac/E) \int_{t=0}^t [\sigma_I(t)^2, \sigma_R(t)^2, \sigma_T(t)^2] dt \quad (A8)$$

The total energy absorbed within the system (i.e. that absorbed during damage generation and minor losses within the fixture) is

$$E_A = E_I - (E_R, E_T), \quad (A9)$$

which can be rewritten as

$$E_A = (Ac/E) \int_{t=0}^t \left\{ \sigma_I(t)^2 - [\sigma_R(t)^2 + \sigma_T(t)^2] \right\} dt \quad (A10)$$

The Hopkinson bars used in this test series were steel with the following parameters:

$$\begin{aligned} A &= 1140 \text{ mm}^2 (1.7675 \text{ in.}^2) \\ c &= 5017 \text{ m s}^{-1} (197531 \text{ in. s}^{-1}) \\ E &= 193.05 \text{ GPa} (28 \times 10^6 \text{ psi}) \end{aligned}$$

so that  $Ac/E = 0.02963 \text{ m}^3 \text{ GPa}^{-1} \text{ s}^{-1} (0.012467 \text{ in.}^5 \text{ lb}^{-1} \text{ s}^{-1})$ .

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestion for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE October 1991		3. REPORT TYPE AND DATES COVERED	
4. TITLE AND SUBTITLE  Energy Absorption of Graphite/Epoxy Plates Using Hopkinson Bar Impact				5. FUNDING NUMBERS  PE: 6.27.84A PR: 4A762784AT42 TA: SS WU: 019 and MIPR FY1456-90-N0066	
6. AUTHORS  Piyush K. Dutta, David Hui and Magna R. Altamirano					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  U.S. Army Cold Regions Research and Engineering Laboratory 72 Lyme Road Hanover, New Hampshire 03755-1290				8. PERFORMING ORGANIZATION REPORT NUMBER  CRREL Report 91-20	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  Office of the Chief of Engineers Washington, D.C. 20314-1000				10. SPONSORING/MONITORING AGENCY REPORT NUMBER  U.S. Air Force Flight Dynamics Laboratory Wright Patterson AFB, Ohio	
11. SUPPLEMENTARY NOTES					
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Approved for public release; distribution is unlimited.  Available from NTIS, Springfield, Virginia 22161				12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  This work summarizes the analytical and experimental study on the energy absorption of quasi-isotropic graphite/epoxy composite plates due to the impacting hemispherical penetrator in a Hopkinson bar apparatus. The mechanics of stress wave propagation through the bar and the composite laminate plate are discussed to predict the phenomenological process involved. The experimental data provided the results in terms of force, velocity, and energy of impact at all times during the penetration process occurring in microseconds. It has been concluded that loss of contact occurs frequently during the penetration process due to the stress wave reflections back and forth in the thickness direction. The damage process seemed to be both velocity and temperature dependent. Below a certain transition velocity the laminate absorbs less energy at low temperature than at high temperature. The reverse is true above this transition velocity. The mechanism of failure tends to change with impact velocity in such a way that progressively less energy is absorbed at higher velocities.					
14. SUBJECT TERMS  Advanced composites    Composites    Fiber composites    Impact    Stress wave Cold    Damage    Graphite/epoxy    Low temperature				15. NUMBER OF PAGES 18	
				16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT  UL		